

The 2012-2013 ANNUAL ASSESSMENT REPORT

Department of Mathematics and Statistics

1. As a result of last year's assessment effort, have you implemented **any changes for your assessment including learning outcomes, assessment plan, assessment tools (methods, rubrics, curriculum map, or key assignment etc.), and/or the university baccalaureate learning goals?**

- a. If so, what are those changes? How did you implement those changes?
- b. How do you know if these changes have achieved the desired results?
- c. If no, why not?

The Department of Mathematics and Statistics has developed a rubric to assess one of the learning outcomes identified to apply to all programs offered by the department. The learning outcome chosen reflects the department's commitment to developing mathematical reasoning in its upper division core courses. The learning outcomes states:

- The mathematics major at CSUS is expected to develop a fundamental understanding of the process and role of mathematics reasoning.

The rubric and the results of the inquiry involving the rubric are given later in this document.

2. As a result of last year's assessment effort, have you implemented **any other changes at the department, the college or the university, including advising, co-curriculum, budgeting and planning?**

- a. If so, what are those changes? How did you implement those changes?
- b. How do you know if these changes have achieved the desired results?
- c. If no, why not?

Due to limited resources and the department's focus on hiring during the past year, the Department of Mathematics and Statistics has not implemented any other changes.

3. What **PROGRAM** (not course) learning outcome(s) have you assessed this academic year?

The Department of Mathematics and Statistics has identified five learning outcomes for all mathematics programs.

1. The mathematics major at CSUS is expected to develop a fundamental understanding of the process and role of mathematics reasoning.
2. The mathematics major at CSUS is expected to develop a fundamental understanding of the main strands of mathematics.
3. The mathematics major at CSUS is expected to have an understanding of and exposure to the breadth of mathematics.
4. The mathematics major at CSUS is expected to demonstrate an ability to effectively communicate mathematical thought.

5. The mathematics major at CSUS is expected to demonstrate a basic understanding of technology and demonstrate the skill to use technology.

For the current assessment cycle the Department of Mathematics and Statistics has chosen to focus on outcomes (1) and to a lesser extent (4) as they relate to the understanding of mathematical reasoning (problem solving) and the ability to effectively communicate mathematical thought.

The process and role of mathematical reasoning is pivotal to all mathematical endeavors. Professional mathematicians regard proof as the intrinsic essence of mathematics, and it is expected that undergraduates will arrive at an appreciation of the role of proof in mathematical discourse, as well as a grasp of the methods of proof that permeate all mathematical exposition.

Students should be able to identify various methods of proof, and apply these methods to their work in upper division courses. Application of these fundamental mathematical methods leads to a deeper insight into the nature of the subject.

The Mathematical Association of America (MAA) report on *Recommendations for the Mathematical Preparation of Teachers of Mathematics* draws our attention to the goal that mathematics teachers must be able to communicate mathematical ideas with ease and clarity. This ability should be expected of all students graduating with a degree in mathematics, and need not be restricted to those planning a career in teaching. The ability to communicate mathematical thought goes to the heart of the mathematical process and centers on the need for clear, logical presentation and exposition.

4. What method(s)/measure(s) have you used to collect the data?

The methods by which the Department of Mathematics and Statistics assesses the effectiveness of its programs are as follows:

1. Math 193 (Capstone Course for the Teaching Credential Candidate)
2. Exam Files for Core Courses
3. Graduate Comprehensive Exam
4. Survey of Alumni
5. Exit Interview

For the current assessment cycle the Department of Mathematics and Statistics has chosen to use the exam files from Math 130A and Math110B.

Math 108 (Introduction to Formal Mathematics), Math 110 (Modern Algebra) and Math 130 (Real Analysis) constitute the central focus of the upper division core for our majors, with all students majoring in mathematics being required to take these sequences. The department is maintaining a file of final exams given in these courses as a means of assessing and reviewing the standard of performance of students completing these courses. These exams are reviewed to ensure that they are comprehensive and examine the areas needed to assess student learning in each of the primary goals of the courses. The Core Curriculum Subcommittee has the responsibility of monitoring the conduct of the core program, and reviews the exam file, current enrollment, and success rates of our students.

5. What are the criteria and/or standards of performance for the program learning outcome?

The Department of Mathematics and Statistics has developed the following Mathematics Reasoning Rubric for determining standards of performance for the mathematical reasoning program learning outcome. This Mathematics Reasoning Rubric is based on the Problem Solving Value Rubric of the Association of American Colleges and Universities which defines problem solving as the process of designing, evaluating and implementing a strategy to answer an open ended question. This definition aligns well with the department's understanding of the process and role of mathematics reasoning.

Mathematics Reasoning Rubric

	Capstone 4	Milestones		Benchmark 1
		3	2	
Understands the Nature of the Problem	Demonstrates a full understanding of all aspects of the problem	Demonstrates an understanding of the problem but may not grasp all aspects involved	Demonstrates an understanding of parts of the problem, but work lacks depth	Demonstrates a limited understanding of any aspect of the problem
Identifies a Strategy for Tackling the Problem	Identifies a clear strategy for solving all aspects of the problem	Identifies a strategy for solving that applies to most, but not all, aspects of the problem	Identifies a strategy that may have limited application	Identifies a strategy that may not apply to any aspect of the problem
Constructs a Valid Solution / Proof for the Problem	Constructs a thorough and logically structured proof that demonstrates full comprehension of the mathematics. Examines the feasibility of the solution	Constructs a logically structured proof, albeit brief or minimal, that demonstrates adequate comprehension of the mathematics	Constructs a limited proof that may contain faulty reasoning and shows minimal understanding of the mathematics	Constructs an incomplete or disorganized proof that may contain faulty reasoning and shows little understanding of the mathematics
Communicates Mathematical Thought and Reasoning	Communicates mathematical ideas effectively with ease and clarity	Communicates mathematical ideas adequately, but not always clearly	Communicates mathematical ideas in a vague or imprecise manner	Communicates mathematical ideas in a vague and imprecise manner that may include inaccuracies

6. What data have you collected? What are the results and findings, including the percentage of students who meet each standard?

- a. In what areas are students doing well and achieving the expectations?

b. In what areas do students need improvement?

The data collected comes from final exams for Math 110B and Math 130A. These courses are part of the Core program for the Department of Mathematics and Statistics. Two questions from each exam are analyzed in terms of the reasoning rubric.

Final exams for Math 130A were chosen to reflect the Learning Outcomes for Math 130. In particular, the learning outcomes examined were:

- Demonstrate an understanding of the formal epsilon-delta definitions of limits, continuity, differentiability and integrability, and be able to establish basic results using these definitions.
- Demonstrate an understanding of the properties of the real numbers, such as finding the infimum and supremum of a set, and using the Archimedean property.

The first question from the Math 130A final considered is:

Suppose $A \subseteq \mathbb{R}$ and $B \subseteq \mathbb{R}$ are non-empty and bounded above. Prove that if $\text{lub}(A) > \text{lub}(B)$, then $\text{lub}(A \cup B) = \text{lub}(A)$.

	Capstone 4	Milestones 3 2		Benchmark 1
Understands the Nature of the Problem	22%	37%	26%	15%
Identifies a Strategy for Tackling the Problem	15%	22%	41%	22%
Constructs a Valid Solution / Proof for the Problem	11%	22%	33%	33%
Communicates Mathematical Thought and Reasoning	11%	15%	41%	33%

The second question from the Math 130A final considered is:

Let $f(x) = \frac{1}{2}x^2$.

- Using the ϵ - δ definition of continuity, prove that f is continuous at any $c \in \mathbb{R}$.
- In the ϵ - δ definition of continuity, find the largest δ that will work for $\epsilon = 1$ at $x = 2$.
- In the ϵ - δ definition of continuity, find the largest δ that will work for $\epsilon = 1$ at $x = 5$.
- (extra credit) Is $f(x)$ uniformly continuous on $\mathbb{R}$? Why or why not?

	Capstone 4	Milestones 3 2		Benchmark 1
Understands the Nature of the Problem	30%	22%	41%	7%
Identifies a Strategy for Tackling the Problem	30%	33%	30%	7%
Constructs a Valid Solution / Proof for the Problem	19%	30%	41%	11%
Communicates Mathematical Thought and Reasoning	7%	33%	48%	11%

Final exams from Math 110B were chosen to reflect the Learning Outcomes for Math 110. The learning outcomes examined were:

- Identify whether a set with multiplicative and additive operations is a ring.
- Identify whether a subset of a ring is an ideal.
- Prove basic results about homomorphisms of rings, and the use the kernel to identify whether a homomorphism is a monomorphism.

The first question from the Math 110B final considered is:

Can a ring have an element which is both a zero divisor and a unit? Prove your answer.

	Capstone 4	Milestones 3 2		Benchmark 1
Understands the Nature of the Problem	59%	9%	23%	9%
Identifies a Strategy for Tackling the Problem	54%	14%	9%	23%
Constructs a Valid Solution / Proof for the Problem	50%	18%	0%	32%
Communicates Mathematical Thought and Reasoning	14%	36%	32%	18%

The second question from the Math 110B final considered is:

Let $\phi: R \rightarrow R'$ be a ring homomorphism, where R' is a commutative ring. Let $a, b \in R$. Prove $ab - ba \in \ker \phi$.

	Capstone 4	Milestones 3 2		Benchmark 1
Understands the Nature of the Problem	55%	14%	23%	9%
Identifies a Strategy for Tackling the Problem	36%	36%	14%	14%
Constructs a Valid Solution / Proof for the Problem	41%	18%	18%	23%
Communicates Mathematical Thought and Reasoning	5%	32%	41%	23%

The most significant discernable feature from the grid is the relatively poor performance of students in communicating mathematical thought. While the numbers in most categories were strong, the efforts of students to express their thinking with ease and clarity needs attention. As stated in the exposition of program learning outcomes, the ability of students to communicate mathematical ideas should be expected of all graduating students since the ability for clear, logical presentation and exposition goes to the heart of the mathematical process. While the numbers in this category of assessment are discouraging, there are several strategies that the Core Curriculum Committee could address as a means of improving this communication skill. Possibilities might include the frequent collection of homework and the timely feedback of work to assist students, as well as instructors focusing on proofs that stress presentation and logic rather than focusing exclusively on the mathematics involved. These are issues to be taken up by the Core Curriculum Committee.

On the positive side, the strong numbers relating to understanding the nature of the problem and identifying a strategy indicates that students have a strong basic conceptual understanding of the mathematics involved and are able to initiate a method of solution. The strength of this conceptual framework means that the students have a solid foundation on which to improve their proof writing skills. The writing of proofs is a talent that takes much work, time and instructor assistance. The ability to write a thorough and logically structured proof is one that evolves as students progress through the mathematics program. From the evidence of the exams considered from Math 130A and Math 110B, the students in the second term of Algebra (Math 110B) demonstrate a greater facility in constructing proofs than students in the first term Analysis (Math 130A) class.

7. As a result of this year's assessment effort, do you anticipate or propose any changes for your program (e.g. structures, content, or learning outcomes)?

a. If so, what changes do you anticipate? How do you plan to implement those changes?

b. How do you know if these changes will achieve the desired results?

The Department of Mathematics and Statistics intends to complete the set of subject learning outcomes for all courses in the Core Program. The department currently has learning outcomes for the Math 110 (Modern Algebra) and Math 130 (Real Analysis) sequences and plans to develop learning outcomes for Math 108 (Introduction to Formal Mathematics). These learning outcomes can then be used in conjunction with the Mathematics Reasoning Rubric to further assess some of the key issues addressed in the current report.

The department also plans to develop additional rubrics in the areas of Critical Thinking and Oral Communication, and hopes to start developing curriculum maps for its program.

8. Which program learning outcome(s) do you plan to assess next year? How?

The Department of Mathematics and Statistics plans to continue the assessment of learning outcomes that were initiated with the current assessment cycle. The department now has in place a Mathematics Reasoning Rubric which can be used to further assess the Core Program. In addition, with the development of learning outcomes for Math 108, greater insight into the needs of the department can be ascertained. The development and implementation of such plans will ultimately be the task of the Assessment Committee for 2013-14.